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Analysis of Natural Vibration of Visco-Elastic Rectangulr Plate with Thickness and Temperature Variation

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Abstracts

Visco- Elastic Plates are being increasingly used in the aeronautical and aerospace industry as well as in other fields of modern technology. Plates with variable thickness are of great importance in a wide variety of engineering applications i.e. nuclear reactor, aeronautical field, naval structure, submarine, earth-quake resistors etc. A mathematical model is presented for the use of engineers and research workers in space technology, have to operate under elevated temperatures.It is assumed that temperature varies exponentially in x-direction :

$$T = T_0\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)$$

and thickness of plate varies parabolic in x & linear in y direction:

$$h = h_0\left(1+\beta_1\frac{x^2}{a^2}\right)\left(1+\beta_2\frac{y}{b}\right)$$

To obtain frequency equation, the Rayleigh-Ritz method is used which allows satisfying all four boundary conditions. The frequency for first two modes of vibration is calculated for different values of thermal gradient and taper constants with the help of latest computational technique i.e. Mat Lab.

Keywords: Vibration, Thermal Gradient, Taper Constant, Frequency, Plate.

Introduction

Vibration means any motion that repeats itself after an interval of time. Some other definitions of vibration as follows:-

1. A rapid linear motion of a particle or of an elastic solid about an equilibrium position.
2. A rapid oscillation of a particle or particles or elastic solid or surface.
3. The particle oscillating about main position with high frequency.
4. Vibrations are mechanical oscillation about a reference position.

Vibrations are classified in to two ways:-

- Desirable / Controlled / Required / Wanted vibrations.
- Undesirable / Uncontrolled / Not required / Unwanted vibrations.

There are many applications in our daily life where vibrations effect are required e.g. in loudspeakers, space shuttles, satellites where discrepancies in the temperature also affect the vibrations. Controlled vibrations effects are required in health industry, paper industry, design of structures, building construction, reducing soil adhesion and many more areas. On the other hand, unwanted vibration causes fatigues. Unwanted vibration can damage electronic components of aerospace system, damage buildings by earthquake, bring tsunami and contribute to toppling of tall smokestacks, collapse of a suspension bridge in a windstorm. Hence, vibrations totally affect our day-to-day life.

Vibrations are of many types such as free vibration, force vibration, linear vibration, non-linear vibration, damped vibration, undamped vibration etc. Free vibrations are those in which energy is neither added nor removed from the vibration system. It will just keep vibrating forever at the same amplitude, whereas, forced vibrations are those in which energy is added to the vibrating system, for example in a clockwork mechanism where the energy stored in a spring is transferred a bit at a time to the vibrating element. Tomar and Tewari [1] presented an analysis on effect of thermal gradient on frequencies of a circular plate of linearly varying thickness. Gupta and Khanna [2-3] discussed the free vibration of clamped visco-elastic rectangular plate having bi-direction thickness variations. Larrondo, Avalos, Laura and Rossi [4] studied vibrations of simply supported rectangular plates with varying thickness and same aspect ratio

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cutouts. Free vibrations of rectangular plates of parabolically varying thickness have been investigated by Jain and Soni [5]. Bhatnagar and Gupta [6] discussed thermal effect on vibration of visco-elastic elliptic plate of variable thickness.
Frequency parameter is calculated for first two modes of vibration for clamped orthotropic rectangular plate whose thickness varies parabolic and linear in both directions under exponentially varying temperature distribution, for various values of thermal gradient α and taper constants β_1, β_2. These results have been compared with those obtained for orthotropic rectangular plate with bi-directional parabolic variation in thickness in the absence of temperature gradient.

Methodology

Let the plate under consideration is subjected to a steady one dimensional temperature distribution T exponentially along x- axis, then as [3]

$$T = T_0\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right) \qquad (1)$$

where T is the temperature excess above the reference temperature at a distance x/a and T_0 is the temperature excess above the reference temperature at the end of the plate i.e. at x=a. For most orthotropic materials, moduli of elasticity (as a function of temperature) are defined as,

$$\left.\begin{aligned} E_x(T) &= E_1\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ E_y(T) &= E_2\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ G_{xy}(T) &= G_0\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \end{aligned}\right\} \qquad (2)$$

Where E_x and E_y are Young's moduli in x- and y-directions respectively and G_{xy} is shear modulus. Here, γ is slope of variation of moduli with temperature. Where $\alpha = \gamma T_0 (0 \le \alpha < 1)$, is thermal gradient parameter.
The governing differential equation of transverse motion of an orthotropic rectangular plate of variable thickness in Cartesian coordinate is,

$$D_x\frac{\partial^4 w}{\partial x^4}+D_y\frac{\partial^4 w}{\partial y^4}+2H\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial H_x}{\partial x}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}+2\frac{\partial H_y}{\partial y}\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial y}$$
$$+2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3}+2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3}+\frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$$
$$+\frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2}+4\frac{\partial^2 D_{xy}}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\rho h\frac{\partial^2 w}{\partial t^2}=0 \qquad (3)$$

Where D_x & D_y are flexural rigidities in x- and y-directions respectively and D_{xy} is torsional rigidity

$$D_1 = \nu_x D_y (= \nu_y D_x) \qquad (4)$$

Where v_x & v_y are Poisson's ratio.
and

$$H = D_1 + 2D_{xy} \qquad (5)$$

For free transverse vibrations of the plate, $w(x, y, t)$ can be defined as,

$$w(x, y, t) = W(x, y)e^{ipt} \qquad (6)$$

where p is radian frequency of vibration.
Two term deflection function for Clamped rectangular plate is taken as,

$$W(x,y) = A_1\left(\frac{x}{a}\right)^2\left(\frac{y}{b}\right)^2\left(1-\frac{x}{a}\right)^2\left(1-\frac{y}{b}\right)^2 + A_2\left(\frac{x}{a}\right)^3\left(\frac{y}{b}\right)^3\left(1-\frac{x}{a}\right)^3\left(1-\frac{y}{b}\right)^3 \qquad (7)$$

Where A_1 and A_2 are constants to be evaluated.
Using equation (3) in the values of D_x, D_y & D_{xy} , we have

$$\begin{aligned} D_x &= \frac{E_1 h^3}{12(1-\nu_x\nu_y)}\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ D_y &= \frac{E_2 h^3}{12(1-\nu_x\nu_y)}\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ D_{xy} &= \frac{G_0 h^3}{12}\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \end{aligned} \qquad (8)$$

When the plate is executing transverse vibration of mode shape W(x,y) then Strain energy V and Kinetic energy T_1 are respectively expressed as,

$$V = \frac{1}{2}\int_0^a\int_0^b\left[\begin{aligned} & D_x\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_1 \times \\ & \left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right) + 4D_{xy}\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2 \end{aligned}\right] dydx \qquad (9)$$

and

$$T_1 = \frac{1}{2}p^2\rho\int_0^a\int_0^b hW^2 dydx \qquad (10)$$

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where ρ is the mass density.

Using equations (4), (7) & (8) in equation (9), we have

$$V = \frac{1}{2}\left(\frac{E_1}{12(1-v_x v_y)}\right)\int_0^a\int_0^b \left\{ \begin{bmatrix} h^3\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\times \\ \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2+\frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial y^2}\right)^2+2v_x\frac{E_2}{E_1} \\ \left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right)+4\frac{G_0}{E_1}(1-v_x v_y)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right]\end{bmatrix}\right\} dydx \quad (11)$$

Variation in Thickness

Thickness h of the plate is assumed to be varying parabolically in both directions

$$h = h_0\left(1+\beta_1\frac{x^2}{a^2}\right)\left(1+\beta_2\frac{y}{b}\right) \quad (4.12)$$

where $h_0 = h|_{\substack{x=0\\y=0}}$

Using equation (4.12) in equations (4.10) & (4.11), we have:

$$T_1 = \frac{1}{2}h_0\rho p^2\int_0^a\int_0^b\left(1+\beta_1\frac{x^2}{a^2}\right)\left(1+\beta_2\frac{y}{b}\right)W^2 dydx$$

and

$$V = \frac{1}{2}\left(\frac{E_1 h_0^3}{12(1-v_x v_y)}\right)\times\int_0^a\int_0^b \left\{ \begin{bmatrix} \left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\times \\ \left(1+\beta_1\frac{x^2}{a^2}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3 \end{bmatrix}\times \begin{bmatrix} \left(\frac{\partial^2 W}{\partial x^2}\right)^2+\frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial y^2}\right)^2+ \\ 2v_x\frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right)+ \\ 4\frac{G_0}{E_1}(1-v_x v_y)\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2 \end{bmatrix}\right\} dydx$$

Method of Solution

An approximate solution to the current problem is given by the application of Rayleigh – Ritz method. In order to apply their procedure, maximum Strain energy must be equal to maximum Kinetic energy. Therefore it is desired that following equation must be satisfied

$$\delta\left(V-\lambda^2 T_1\right)=0 \quad (15)$$

On substituting the values of 'V' & 'T_1' from equations (14) & (13) in equation (15), we have

$$\delta\left(V_1-\lambda^2 T_2\right)=0 \quad (16)$$

where

$$\lambda^2 = \frac{12a^4\rho\, p^2(1-v_x v_y)}{E_1 h_0^2} \quad (17)$$

is a frequency parameter ,

$$V_1 = \int_0^a\int_0^b \left\{ \begin{bmatrix} \left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right]\times \\ \left(1+\beta_1\frac{x^2}{a^2}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3 \end{bmatrix}\times \begin{bmatrix} \left(\frac{\partial^2 W}{\partial x^2}\right)^2+\frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial y^2}\right)^2+ \\ 2v_x\frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right)+ \\ 4\frac{G_0}{E_1}(1-v_x v_y)\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2 \end{bmatrix}\right\} dydx$$

and

$$T_2 = \int_0^a\int_0^b\left(1+\beta_1\frac{x^2}{a^2}\right)\left(1+\beta_2\frac{y}{b}\right)W^2 dydx \quad (19)$$

Boundary Condition and Frequency Equation

For a clamped rectangular plate, boundary conditions are,

$$W = \frac{\partial W}{\partial x} = 0 \text{ at } x = 0, a$$
$$W = \frac{\partial W}{\partial y} = 0 \text{ at } x = 0, a \quad (20)$$

Equation (16) contains two unknown parameters A_1 & A_2 to be evaluated. Values of these constants may be evaluated by the following procedure

$$\frac{\partial}{\partial A_q}\left(V_1-\lambda^2 T_2\right)=0 \quad where,\ q=1,2 \quad (21)$$

On simplifying equation (21), we get following form,

$$c_{q1}A_1 + c_{q2}A_2 = 0 \quad (22)$$

where c_{q1} & c_{q2} involves the parametric constants and the frequency parameter.

For a non - zero solution, determinant of coefficients of equation (22) must vanish. In this way frequency equation comes out to be:

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0 \quad (23)$$

Frequency equation (23) is quadratic in λ^2, so it will give two roots. These two values represent the two modes of vibration of frequency . From equation (23) one can easily obtain frequency for both the mode.

Numerical evaluations

Frequency parameter is calculated for the first two modes of vibration for different values of taper constants and thermal gradient, for a clamped plate with parabolically variation in thickness in both directions. Frequency equation (14) is quadratic in λ^2, so will give two roots. Let they be λ_1 & λ_2 respectively. These two values correspond to first and second modes of vibration respectively. The parameter for orthotropic material has been taken as:

$$\frac{E_2}{E_1} = 0.32 \quad , v_x \frac{E_2}{E_1} = 0.04,$$

$$\frac{G_0}{E_1}(1 - v_x v_y) = 0.09$$

Verification of work is done by comparing these with unheated plate for doing so thermal gradient and taper constants are allowed to be zero.

Results and discussion

Table 1 shows variation of frequency parameter λ with thermal gradient parameter 'α', for various values of taper constants β_1 & β_2, for a clamped plate for both modes of vibration. From fig. it is clear that with increase in 'α', decrease whether β_1 & β_2 are zero or non-zero. It is to be noticed that λ decreases sharply for second mode of vibration as compared to the first mode of vibration.

Table 2 display the variation of taper constant 'β_1' with frequency parameter λ, for both modes of vibration. It is observed that for both the modes of vibration λ increases with increase in 'β_1', whether the plate is heated or unheated. For non-zero 'β_2', 'λ' has higher value and from unheated to heated plate, value of 'λ' decrease.

A comparative study was done for the plates regarding variation in thickness under exponential temperature gradient i.e. plates with linear and parabolic variations in thickness were compared. It was found that for plate having parabolic bi-directional variation in thickness, all the effects were occurring for lesser values of frequency parameter as compared to that of linear bi-directional variation in thickness. Hence it is concluded that plates with parabolic variation in thickness are more stable as compared to those of linearly varying thickness, for bearing up of exponential thermal gradient effects.

Table 1: ***Values of frequency parameter λ^2 for different values of thermal gradient α* and constant aspect ratio a/b = 1.5**

α	$\beta_1 = \beta_2 = 0.0$		$\beta_1=0.2$, $\beta_2=0.6$	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**
0.0	37.100125	151.989765	50.259234	204.088750
0.2	36.375345	143.127576	47.174914	195.000857
0.4	34.156701	133.286754	44.120149	182.001400
0.6	29.478911	122.011026	40.191510	168.280211
0.8	26.298461	110.105108	36.226361	154.123412
1.0	23.301244	99.173126	32.170169	142.291123

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Table 2: ***Values of frequency parameter λ^2 for different values of taper constant β_1 with thermal gradient α=0.0 and constant aspect ratio a/b = 1.5***

β_1	β_2= **0.0**		β_2=**0.6**	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**
0.0	37.100122	151.182762	47.130832	192.220501
0.2	39.547602	162.357323	50.129232	204.088711
0.4	42.621372	174.130581	53.229124	219.141731
0.6	45.351001	186.139346	57.309312	234.195433
0.8	49.174634	200.121068	61.231782	252.250523
1.0	51.134173	214.115691	65.221893	265.168244

Conclusion

Aim is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field & increase strength, durability and efficiency of mechanical design and structuring with a practical approach .Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers practitioners.Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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